Name:

Exam Style Questions

Algebraic Proof

Corbettmaths

Equipment needed: Pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Video Tutorial

www.corbettmaths.com/contents

Video 365

Answers and Video Solutions

1. Prove that the sum of three consecutive integers is divisible by 3.

three consecutive integers ; n
$n+1$
$n+2$

$n+(n+1)+(n+2) = 3n+3$
$= 3(n+1)$
$\therefore$ divisible by 3

(3)

2. Prove $(n+6)^2 - (n+2)^2$ is always a multiple of 8

$(n+6)(n+6) = n^2 + 12n + 36$
$(n+2)(n+2) = n^2 + 4n + 4$

$n^2 + 12n + 36 - n^2 - 4n - 4$
$= 8n + 32$
$= 8(n+4)$
$\therefore$ divisible by 8 and is therefore a multiple of 8

(4)

3. Prove $(n + 10)^2 - (n + 5)^2$ is always a multiple of 5

$(n+10)(n+10) = n^2 + 20n + 100$

$(n+5)(n+5) = n^2 + 10n + 25$

sub:

$10n + 75$

$= 5(2n + 15)$

$\therefore$ a multiple of 5

(4)

4. Prove the sum of two consecutive odd numbers is even.

two consecutive odd numbers $2n+1$, $2n+3$

Sum of $2n+1$ & $2n+3$ $= 4n+4$

$4(n+1)$

$\therefore$ an even number.

(3)

5. Prove $(2n + 1)(3n - 2) - (6n - 1)(n - 2)$ is always even

$$6n^2 - 4n + 3n - 2 - (6n^2 - 12n - n + 2)$$

$$6n^2 - n - 2 - (6n^2 - 13n + 2)$$

$$12n - 4$$

$$= 4(3n - 1)$$

$\therefore$ even.

(3)

6. Prove that the sum of three consecutive even numbers is always a multiple of 6

Three consecutive even numbers $2n$

$2n + 2$

$2n + 4$

Sum

$$2n + (2n + 2) + (2n + 4)$$

$$= 6n + 6$$

$$= 6(n + 1)$$

$\therefore$ multiple of 6.

(3)

7. Prove the sum of four consecutive odd numbers is always a multiple of 8

four consecutive odd numbers ; $2n+1$

$2n+3$

$2n+5$

$2n+7$

Sum of

$(2n+1)+(2n+3)+(2n+5)+(2n+7)$

$= 8n+16$

$= 8(n+2)$

$\therefore$ a multiple of 8.

(4)

8. Prove $(2n+9)^2-(2n+5)^2$ is always a multiple of 4

$(2n+9)(2n+9) - (2n+5)(2n+5)$

$= 4n^2+36n+81 - (4n^2+20n+25)$

$= 16n+56$

$= 4(4n+14)$

$\therefore$ a multiple of 4.

(4)

9. Prove $(n+1)^2 + (n+3)^2 - (n+5)^2 \equiv (n+3)(n-5)$

$(n+1)(n+1) + (n+3)(n+3) - (n+5)(n+5)$

$= n^2 + 2n + 1 + (n^2+6n+9) - (n^2 + 10n + 25)$

$= 2n^2 + 8n + 10 - n^2 - 10n - 25$

$= n^2 - 2n - 15$

$= (n+3)(n-5)$

(4)

10. Prove the product of two even numbers is always even

Even numbers : $2n$ & $2m$

product : $2n \times 2m = 4mn$

$2(2mn)$

$\therefore$ always even.

(3)

11. Prove the product of three consecutive odd numbers is odd

Three consecutive odd numbers : $2n+1$

$2n+3$

$2n+5$

Product : $(2n+1)(2n+3)(2n+5)$

$8n^3 + 36n^2 + 46n + 15$

$\underbrace{2(4n^3 + 18n^2 + 23n)}_{\text{even}} + 15$ + odd

even + odd = odd

(3)

12. Prove algebraically that the sum of the squares of two odd integers is always even.

$2n+1$ and $2k+1$

$(2n+1)^2 + (2k+1)^2$

$4n^2 + 4n + 1 + 4k^2 + 4k + 1$

$= 4n^2 + 4n + 4k^2 + 4k + 2$

$= 2(2n^2 + 2n + 2k^2 + 2k + 1)$

$\therefore$ <u>even</u>

(4)

13. Prove that when two consecutive integers are squared, that the difference is equal to the sum of the two consecutive integers.

Two consecutive integers : n

$n+1$

Sum $n+n+1 = 2n+1$

Difference between squares :

$= (n+1)^2 - n^2$

$= n^2 + 2n + 1 - n^2$

$= 2n+1$

$\therefore$ they are equal.

(4)

14. Given x is a ~~n~~ positive integer.

Prove that $(2x+3)^2 - 3x(x+2)$ is a square number.

$(2x+3)(2x+3)$

$4x^2 + 6x + 6x + 9 - 3x^2 - 6x$

$4x^2 + 12x + 9 - 3x^2 - 6x$

$x^2 + 6x + 9$

$(x+3)(x+3)$

$= (x+3)^2$

$\therefore$ a square number.

(3)

15. Prove that the sum of the squares of 3 consecutive positive integers is always 1 less than a multiple of 3.

$n \quad , \quad n+1 \quad , \quad n+2$

n^2

$(n+1)^2 = n^2 + 2n + 1$

$(n+2)^2 = n^2 + 4n + 4$

$n^2 + (n^2 + 2n + 1) + (n^2 + 4n + 4)$

$= 3n^2 + 6n + 5$

$3n^2 + 6n + 6$ is a multiple of 3 since $3(n^2 + 2n + 2)$

$\therefore$ $3n^2 + 6n + 5$ is 1 less than a multiple of 3.

(4)

16. Prove algebraically that

$$(4n + 1)^2 - (2n - 1) \text{ is an even number}$$

for all positive integer values of n.

$(4n+1)(4n+1) - (2n-1)$

$= 16n^2 + 8n + 1 - 2n + 1$

$= 16n^2 + 6n + 2$

$= 2(8n^2 + 3n + 1)$

$\therefore$ even

(4)

$(n-6)(n-6)$

17. Prove that $3n(3n + 4) + (n - 6)^2$ is positive for all values of n

$9n^2 + 12n + (n^2 - 12n + 36)$

$10n^2 + 36$

$n^2 \geq 0$

$10n^2 \geq 0$

$10n^2 + 36 > 0$

$\therefore$ $10n^2 + 36$ is always positive.

(4)

18. The first five terms of a linear sequence are 5, 11, 17, 23, 29 …

$6n$ 6 12 18 24 30

(a) Find the nth term of the sequence

$6n - 1$

(2)

A new sequence is generated by squaring each term of the linear sequence and then adding 5.

(b) Prove that all terms in the new sequence are divisible by 6.

$(6n-1)^2 + 5$

$(6n-1)(6n-1) + 5$

$36n^2 - 12n + 6$

$6(6n^2 - 2n + 1)$ $\therefore$ divisible by 6

(4)

19. Prove that the product of two consecutive even numbers is a multiple of 4.

Two consecutive even numbers : $2n$
$2n + 2$

product : $2n(2n+2)$
$4n^2 + 4n$

$4n(n+1)$

$\therefore$ a multiple of 4.

(3)

20. Prove that when any odd integer is squared, the result is always one more than a multiple of 8.

odd integer : $2n + 1$

odd integer squared : $(2n+1)(2n+1)$
$= 4n^2 + 4n + 1$
$= 4n(n+1) + 1$

since either n or $n+1$ is even,
that means $n(n+1)$ is even.
4 times even is always a multiple of 8.

$\therefore$ $4n(n+1)+1$ is one more than a multiple of 8.

(4)

21. Prove that the product of two odd numbers is always odd.

odd numbers : $2k+1$

$2m+1$

product $(2k+1)(2m+1)$

$4km + 2k + 2m + 1$

$\underbrace{2(2km + k + m)}_{\text{even}} + 1$

even + odd = odd

(3)

22. Prove algebraically that every term in the sequence $n^2 - 12n + 38$ is positive.

$(n-6)^2 - 36 + 38$

$(n-6)^2 + 2$

Since $(n-6)^2 \geq 0$

$(n-6)^2 + 2 > 0$

(3)

23. Prove that the sum of the cubes of two consecutive odd integers is always a multiple of 4.

$2n+1$ and $2n+3$

$(2n+1)^3$
$= (2n+1)(2n+1)(2n+1)$
$= (4n^2+4n+1)(2n+1)$
$= 8n^3+12n^2+6n+1$

$(2n+3)^3$
$= (2n+3)(2n+3)(2n+3)$
$= (4n^2+12n+9)(2n+3)$
$= 8n^3+36n^2+54n+27$

$16n^3+48n^2+60n+28$

$4(4n^3+12n^2+15n+7)$

$\therefore$ multiple of 4

(4)