Name:

Exam Style Questions

Iteration

Corbettmaths

Ensure you have: Pencil, pen, ruler, protractor, pair of compasses and eraser

You may use tracing paper if needed

Guidance

1. Read each question carefully before you begin answering it.
2. Don't spend too long on one question.
3. Attempt every question.
4. Check your answers seem right.
5. Always show your workings

Revision for this topic

www.corbettmaths.com/contents

Video 373

1. The table below shows values of x and y for $y = x^3 - 8x - 10$

x	0	1	2	3	4
y	-10	-17	-18	-7	22

Between which two consecutive integers is there a solution to the equation $x^3 - 8x - 10 = 0$?

Explain your answer.

x = ...3.... and x = ...4....

when $x = 3$ $3^3 - 8 \times 3 - 10 = -7$

when $x = 4$ $4^3 - 8 \times 4 - 10 = 22$

Since there is a change in sign, there will be a solution

(2)

2. Using $x_{n+1} = 8 - \dfrac{5}{x_n^2}$

with $x_0 = 1$

find the values of x_1, x_2, x_3 and x_4

$x_1 = 8 - \frac{5}{1^2} = 3$

$x_2 = 8 - \frac{5}{3^2} = 7.444...$

$x_3 = 8 - \frac{5}{7.444..^2} = 7.909779461$

$x_4 = 8 - \frac{5}{7.9097..^2} = 7.920082617$

x_1 = ...3....

x_2 = ...$7.\dot{4}$....

x_3 = ...7.909779461

x_4 = ...7.920082617

(4)

3. Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{8}{9} - \frac{x_n^3}{9}$ three times to find an estimate for the solution of $x^3 + 9x = 8$

$x_1 = \frac{8}{9} - \frac{0^3}{9} = 8/9$

$x_2 = \frac{8}{9} - \frac{(0.\dot{8})^3}{9} = 0.8108520043$

$x_3 = \frac{8}{9} - \frac{(0.8108..)^3}{9} = 0.8296533595$

0.8296533595
....
(3)

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4. Which of the following iteration formulae cannot be found by rearranging the equation $x^2 - 9x + 2 = 0$?

A $\boxed{x_{n+1} = 9 - \frac{2}{x_n}}$

B ✓ $\boxed{x_{n+1} = \frac{x_n^2}{9} + \frac{2}{9}}$

C $\boxed{x_{n+1} = \frac{9}{2} - \frac{x_n}{2}}$

D ✓ $\boxed{x_{n+1} = \sqrt{9x_n - 2}}$

$x^2 = 9x - 2$

$x = \sqrt{9x - 2}$

$\therefore$ D is possible

$x^2 + 2 = 9x$

$9x = x^2 + 2$

$x = \frac{x^2}{9} + \frac{2}{9}$

$\therefore$ B is possible

$x^2 = 9x - 2$

$x = 9 - \frac{2}{x}$

$\therefore$ A is possible

C
....
(3)

5. (a) Show that the equation $x^3 + 2x = 1$ has a solution between $x = 0$ and $x = 1$

$x^3 + 2x - 1 = 0$

when

$x = 0$ $\quad 0^3 + 2 \times 0 - 1 = -1$

$x = 1$ $\quad 1^3 + 2 \times 1 - 1 = 2$

Since there is a change in sign between $x = 0$ & $x = 1$ there is a solution

(2)

(b) Show that the equation $x^3 + 2x = 1$ can be rearranged to give $x = \dfrac{1}{2} - \dfrac{x^3}{2}$

$2x = 1 - x^3$

$x = \dfrac{1}{2} - \dfrac{x^3}{2}$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{1}{2} - \dfrac{x_n^3}{2}$ twice to find an estimate for the solution of $x^3 + 2x = 1$

$x_1 = \dfrac{1}{2} - \dfrac{0^3}{2} = 0.5$

$x_2 = \dfrac{1}{2} - \dfrac{0.5^3}{2} = \underline{0.4375}$

(3)

6. (a) Show that the equation $3x - x^3 = -11$ has a solution between $x = 2$ and $x = 3$

$3x - x^3 + 11 = 0$

when $x = 2$ $\quad 3 \times 2 - 2^3 + 11 = 9$

$x = 3$ $\quad 3 \times 3 - 3^3 + 11 = -7$

Since there is a change of sign between $x = 2$ & $x = 3$ there is a solution to $3x - x^3 + 11 = 0$ between $x = 2$ & $x = 3$

(2)

(b) Show that the equation $3x - x^3 = -11$ can be rearranged to give

$$x = \sqrt[3]{3x + 11}$$

$3x + 11 = x^3$

$\sqrt[3]{3x + 11} = x$

(2)

(c) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 11}$ three times to find an estimate for the solution of $3x - x^3 = -11$

$x_1 = \sqrt[3]{(3 \times 3) + 11} = 2.714417617$

$x_2 = \sqrt[3]{(3 \times 2.714...) + 11} = 2.675091113$

$x_3 = \sqrt[3]{(3 \times 2.675..) + 11} = 2.669584272$

(3)

7. Using $x_{n+1} = -3 - \dfrac{2}{x_n^2}$

with $x_0 = -3.5$

(a) find the values of x_1 , x_2 and x_3

$x_1 = -3 - \dfrac{2}{(-3.5)^2} = -3.163265306$

$x_2 = -3 - \dfrac{2}{(-3.163..)^2} = -3.199875130$

$x_3 = -3 - \dfrac{2}{(-3.1998..)^2} = -3.195327744$

$x_1 =$ -3.163265306

$x_2 =$ -3.199875130

$x_3 =$ -3.195327744

(3)

(b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^3 + 3x^2 + 2 = 0$

x_1, x_2 and x_3 are increasingly accurate estimates to one of the roots of the equation $x^3 + 3x^2 + 2 = 0$

(2)

8. (a) Show that the equation $20 - x^3 - 7x^2 = 0$ can be rearranged to give

$$x = \frac{20}{x^2} - 7$$

$$20 - 7x^2 = x^3$$

$$\frac{20}{x^2} - 7 = x$$

$$x = \frac{20}{x^2} - 7$$

(2)

(b) Using $x_{n+1} = \frac{20}{x_n^2} - 7$ with $x_0 = -9$

find the values of x_1, x_2 and x_3

$$x_1 = \frac{20}{(-9)^2} - 7 = -6.75308642$$

$$x_2 = \frac{20}{(-6.75..)^2} - 7 = -6.561443673$$

$$x_3 = \frac{20}{(-6.56..)^2} - 7 = -6.535451368$$

x_1 = -6.75308642

x_2 = -6.561443673

x_3 = -6.535451368

(3)

(b) Explain what the values of x_1, x_2 and x_3 represent

x_1, x_2 and x_3 are increasingly accurate estimates to a solution of $20 - x^3 - 7x^2 = 0$

(2)

9. Below are three iteration formulae to find approximation solutions to the equation $6x - x^2 - 7 = 0$

Also shown are three possible values for x_0

Match each iterative formula to a suitable x_0 so that each formula gives an approximate solution to the equation $6x - x^2 - 7 = 0$

You may only use each value of x_0 once

A $\quad x_{n+1} = \dfrac{x_n^2}{6} + \dfrac{7}{6}$ $\qquad x_0 = 1.2 \quad$ 1

B $\quad x_{n+1} = \sqrt{6x_n - 7}$ $\qquad x_0 = 0 \quad$ 2

C $\quad x_{n+1} = 6 - \dfrac{7}{x_n}$ $\qquad x_0 = 2 \quad$ 3

$A \Rightarrow 2$

(3)

since B is not possible $x_1 = \sqrt{6 \times 0 - 7}$ when $x_0 = 0$

$= \sqrt{-7}$

also C is not possible $x_1 = 6 - \frac{7}{0}$ when $x_0 = 0$

$B \Rightarrow 3$

since $x_1 = \sqrt{6 \times 1.2 - 7} = \sqrt{0.2}$

$x_2 = \sqrt{6 \times 0.447.. - 7} = \sqrt{-4.316..}$ not possible when $x_0 = 1.2$

$C \Rightarrow 1$

10. (a) Show that the equation $x^4 - 5x + 1 = 0$ has a root between x = 1.5 and x = 2

when $x = 1.5$

$1.5^4 - 5 \times 1.5 + 1 = -1.4375$

when $x = 2$

$2^4 - 5 \times 2 + 1 = 7.$

Since there is a change of sign $x^4 - 5x + 1 = 0$ has a solution between $x = 1.5$ & $x = 2$

(2)

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{5 - \frac{1}{x_n}}$ three times with $x_0 = 1.5$

to find an estimate for the solution of $x^4 - 5x + 1 = 0$

$x_1 = \sqrt[3]{5 - \frac{1}{1.5}} = 1.630324416$

$x_2 = \sqrt[3]{5 - \frac{1}{1.630..}} = 1.636980507$

$x_3 = \sqrt[3]{5 - \frac{1}{1.6369..}} = 1.637290685$

....

(3)

11. The equation $x^3 - 2x^2 + 19 = 0$ has a root in the interval $(-3, -2)$

Use an appropriate iteration formula to find an approximate to 2 decimal places for the root of $x^3 - 2x^2 + 19 = 0$ in the interval $(-3, -2)$

$$x^3 = 2x^2 - 19$$

$$x = \sqrt[3]{2x^2 - 19}$$

$$x_{n+1} = \sqrt[3]{2(x_n)^2 - 19}$$

$x_0 = -2$

$x_1 = -2.223980091$

$x_2 = -2.08835773$

$x_3 = -2.174183353$

$x_4 = -2.121313821$

$x_5 = -2.154438665$

$x_6 = -2.133900886$

$x_7 = -2.146718196$

$x_8 = -2.138751563$

$x_9 = -2.143715813$

$x_{10} = -2.140627311$

-2.14 ....

(5)