Name:

Exam Style Questions

Completing the Square

Corbettmaths

Equipment needed: Pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Video Tutorial

www.corbettmaths.com/contents

Videos 10, 267a

Answers and Video Solutions

1. Write $x^2 + 8x + 6$ in the form $(x + a)^2 + b$, where a and b are constants.

$(x+4)^2 - 16 + 6$

$(x+4)^2 - 10$

$(x+4)^2 - 10$

(2)

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2. Write $x^2 + 12x - 1$ in the form $(x + a)^2 + b$, where a and b are constants.

$(x+6)^2 - 36 - 1$

$(x+6)^2 - 37$

$(x+6)^2 - 37$

(2)

3. Express $x^2 - 4x - 9$ in the form $(x - a)^2 - b$

$(x-2)^2 - 4 - 9$

$(x-2)^2 - 13$

$(x-2)^2 - 13$

(2)

4. Express $x^2 + 6x + 25$ in the form $(x + a)^2 + b$

$(x+3)^2 - 9 + 25$

$(x+3)^2 + 16$

$(x+3)^2 + 16$

(2)

5. Roz has been asked to write $x^2 + 10x + 7$ in the form $(x + a)^2 - b$

Here is her working out.

$x^2 + 10x + 7$

$(x + 10)^2 - 100 + 7$

$(x + 10)^2 - 93$

Is Roz correct?
Explain your answer.

$(x+5)^2 - 25 + 7$

No - it should be $(x+5)^2 - 18$

No, Roz should have halved 10, to give $(x+5)^2$, also it should be $-25+7$, not $-100+7$

(2)

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6. Write $x^2 - 3x + 7$ in the form $(x + a)^2 + b$

$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 7$

$\frac{7}{1} = \frac{28}{4}$

$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{28}{4}$

$\left(x - \frac{3}{2}\right)^2 + \frac{19}{4}$

$\left(x - \frac{3}{2}\right)^2 + \frac{19}{4}$

(3)

7. Georgina rewrites the expression $x^2 + px + q$ by completing the square.
She correctly obtains $(x - 5)^2 + 31$

Work out the values of p and q.

$(x-5)(x-5)+31$

$x^2 - 5x - 5x + 25 + 31$

$x^2 - 10x + 56$

p = -10 and q = 56

(3)

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8. $x^2 - 4x + b \equiv (x + a)^2 + 11$

Work out the values of a and b

$a = -2$

$(x-2)^2 + 11$

$(x-2)^2 + 11$

$(x-2)(x-2) + 11$

$x^2 - 2x - 2x + 4 + 11$

$x^2 - 4x + 15$

a = -2 b = 15

(3)

9. $x^2 + 5ax + b \equiv (x + 20)^2 - 3a$

Work out the values of a and b

$20 \times 2 = 40$

$5a = 40$

$a = 8$

$(x+20)(x+20) - 24$

$x^2 + 20x + 20x + 400 - 24$

$x^2 + 40x + 376$

$a =$ 8 $b =$ 376

(3)

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10. $x^2 - 6x - 3 \equiv (x - a)^2 - b$, where a and b are constants

(a) Find the values of a and b.

$(x-3)^2 - 9 - 3$

$(x-3)^2 - 12$

a = 3 and b = 12

(3)

(b) Hence solve $x^2 - 6x - 3 = 0$

$(x-3)^2 - 12 = 0$

$(x-3)^2 = 12$

$x - 3 = \pm\sqrt{12}$

$x = 3 \pm \sqrt{12}$

$x = 3 + \sqrt{12}$ $\quad x = 3 - \sqrt{12}$

$x = 3 + (\sqrt{4} \times \sqrt{3})$ $\quad x = 3 - (\sqrt{4} \times \sqrt{3})$

$x = 3 + 2\sqrt{3}$ $\quad x = 3 - 2\sqrt{3}$

x = $3+2\sqrt{3}$ or x = $3-2\sqrt{3}$

(3)

11. Using completing the square, solve $x^2 - 6x + 2 = 0$

$(x-3)^2 - 9 + 2 = 0$

$(x-3)^2 - 7 = 0$

$(x-3)^2 = 7$

$x - 3 = \pm\sqrt{7}$

$x = 3 \pm \sqrt{7}$

$x = 3 + \sqrt{7}$ or $x = 3 - \sqrt{7}$

(5)

12. Using completing the square, solve $x^2 + 4x + 1 = 0$
Give your answers in surd form.

$(x+2)^2 - 4 + 1 = 0$

$(x+2)^2 - 3 = 0$

$(x+2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x = -2 \pm \sqrt{3}$

$x = -2 + \sqrt{3}$ or $x = -2 - \sqrt{3}$

(5)

13. Using completing the square, solve $x^2 - 14x - 2 = 0$

Give your answers in surd form.

$(x-7)^2 - 49 - 2 = 0$

$(x-7)^2 - 51 = 0$

$(x-7)^2 = 51$

$x - 7 = \pm\sqrt{51}$

$x = 7 \pm \sqrt{51}$

$x = 7 + \sqrt{51}$ or $x = 7 - \sqrt{51}$

$x = 7 + \sqrt{51}$ or $x = 7 - \sqrt{51}$

(5)

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14. Express $3x^2 + 18x - 1$ in the form $a(x+b)^2 + c$

$3(x^2 + 6x) - 1$

$3[(x+3)^2 - 9] - 1$

$3(x+3)^2 - 27 - 1$

$3(x+3)^2 - 28$

or

$3(x^2 + 6x - \frac{1}{3})$

$3[(x+3)^2 - 9 - \frac{1}{3}]$

$3[(x+3)^2 - \frac{27}{3} - \frac{1}{3}]$

$3[(x+3)^2 - \frac{28}{3}]$

$3(x+3)^2 - 28$

$3(x+3)^2 - 28$

(3)

15. Write $3x^2 - 12x + 4$ in the form $a(x + b)^2 + c$, where a, b and c are constants

$3(x^2 - 4x) + 4$

$3[(x-2)^2 - 4] + 4$

$3(x-2)^2 - 12 + 4$

$3(x-2)^2 - 8$

or

$3[x^2 - 4x + \frac{4}{3}]$

$3[(x-2)^2 - 4 + \frac{4}{3}]$

$3[(x-2)^2 - \frac{12}{3} + \frac{4}{3}]$

$3[(x-2)^2 - \frac{8}{3}]$

$3(x-2)^2 - 8$

$3(x-2)^2 - 8$

(4)

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16. Use completing the square to find the minimum point of the curve $y = x^2 - 6x + 1$

$y = (x-3)^2 - 9 + 1$

$y = (x-3)^2 - 8$

x^2 translated 3 squares right & 8 squares down.

$(3, -8)$

(4)

17. Use completing the square to find the minimum point of the curve $y = x^2 + 4x + 7$

$y = (x+2)^2 - 4 + 7$

$y = (x+2)^2 + 3$

x^2 translated 2 squares left and 3 squares up.

(-2, 3)

(4)

18. A curve has equation $y = x^2 - 10x + 20$

(a) Write $x^2 - 10x + 20$ in the form $(x-a)^2 - b$

$(x-5)^2 - 25 + 20$

$(x-5)^2 - 5$

$(x-5)^2 - 5$

(3)

(b) Write down the equation of the line of symmetry of $y = x^2 - 10x + 20$

x^2 translated 5 right

$x = 5$

(1)

19. (a) Write $x^2 + 8x - 7$ in the form $(x + a)^2 - b$

$(x+4)^2 - 16 - 7$

$(x+4)^2 - 23$

$(x+4)^2 - 23$

(2)

(b) Solve the equation $x^2 + 8x - 7 = 0$

Give your answers in surd form.

$(x+4)^2 - 23 = 0$

$(x+4)^2 = 23$

$x+4 = \pm\sqrt{23}$

$x = -4 \pm \sqrt{23}$

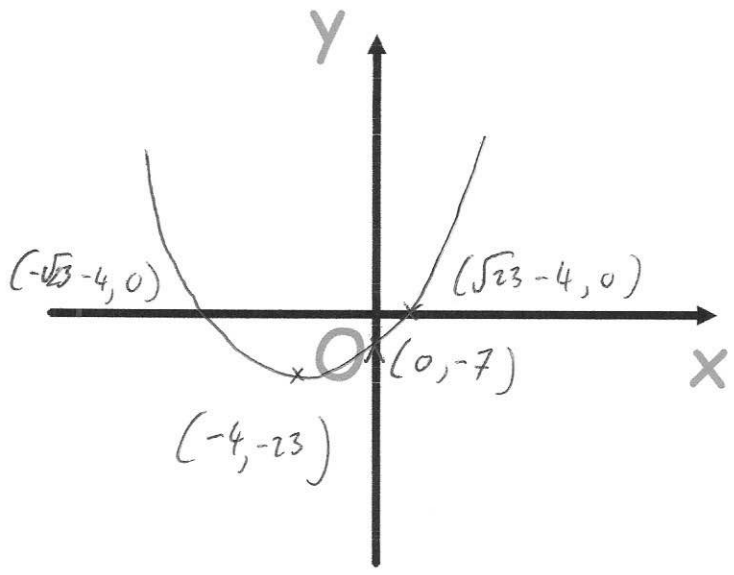
$-4 + \sqrt{23}$ or $-4 - \sqrt{23}$

x = $\sqrt{23} - 4$ and x = $-\sqrt{23} - 4$

(3)

(c) Sketch the graph of $y = x^2 + 8x - 7$

Show the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.



(4)

20. The nth term of a sequence is $n^2 - 6n + 13$

By using completing the square, show that every term is positive.

$(n-3)^2 - 9 + 13$

$(n-3)^2 + 4$

$(n-3)^2$ is always greater than or equal to zero.

$(n-3)^2 \geq 0$

$\therefore$

$(n-3)^2 + 4$ is always positive.

QED

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